

FAITHFULLY QUADRATIC RINGS

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The theory of *special groups*, [1], was initially conceived as an abstract framework embodying as particular cases various theories of quadratic forms over fields. This abstract framework was employed in solving some outstanding open problems in quadratic form theory over formally real fields.

Early on M. Knebusch observed that this set up might be used to investigate theories of quadratic forms over certain classes of (commutative, unitary) rings. This talk will present the main ideas and results of such investigation, initiated by the authors in 2004/2005, and now published *in extenso* in [2].

Our results deal with diagonal quadratic forms with invertible coefficients over rings meeting some minimal conditions of orderability. The approach is based on an extension of the classical notion of isometry of quadratic forms to preordered rings (**p-rings**) (A, T) , where 2 is invertible, bringing the preorder T into play. We give three conditions in terms of this notion and the related notion of representation which are sufficient — and, under mild restrictions, also necessary — to guarantee that the intrinsic theory of quadratic forms in (A, T) coincides with the formal theory in the canonically associated (special) group $G_T(A)$.

It follows that p-rings satisfying these axioms possess a rich theory of diagonal quadratic forms with invertible coefficients, previously known to hold only in the case of formally real fields. The scheme above also applies to the case where $T = A^2$ and -1 is not a sum of squares in A .

Examples of p-rings satisfying our axioms are:

- (A large class of) rings with many units (for all preorders and the squares).
- Reduced f -rings (including the outstanding case of rings of continuous real-valued functions on a topological space).
- Strictly representable rings (including the case of real holomorphy rings of formally real fields).

References.

[1] M. Dickmann, F. Miraglia, **Special Groups. Boolean-Theoretic Methods in the Theory of Quadratic Forms**, Memoirs AMS **689** (2000), 247 pp.

[2] M. Dickmann, F. Miraglia, **Faithfully Quadratic Rings**, Memoirs AMS, to appear 2014, 160 pp.

This is joint work with F. Miraglia (University of São Paulo, Brazil)