ON INVERSE LIMITS OF CIRCULAR UNITS IN CYCLOTOMIC $\mathbb{Z}_p$-EXTENSIONS OF REAL ABELIAN FIELDS

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Let p be a prime number and let $K_\infty = \bigcup_{n=0}^{\infty} K_n$ be the cyclotomic $\mathbb{Z}_p$-extension of a real abelian field K. Let $C(K_n)$ and $\overline{C}(K_n)$ be the Sinnott's and Washington's group of circular units of K_n, respectively. Let $C(K_\infty) = \varprojlim(C(K_n) \otimes \mathbb{Z}_p)$ and $\overline{C}(K_\infty) = \varprojlim(\overline{C}(K_n) \otimes \mathbb{Z}_p)$ be the inverse limits with respect to norms. In 1993 Kučera and Nekovář proved that, for $p \neq 2$, the index of $C(K_\infty)$ in $\overline{C}(K_\infty)$ is finite. In 2002 Belliard found an infinite family of real abelian fields K satisfying $C(K_\infty) \neq \overline{C}(K_\infty)$ by a construction of an explicit element of $\overline{C}(K_\infty) - C(K_\infty)$. Recently Bulant and Kučera proposed a definition of another group $C'(K)$ of circular units of a real abelian field K satisfying $C(K) \subseteq C'(K) \subseteq \overline{C}(K)$. Similarly as $C(K)$, the group $C'(K)$ is generated by explicit generators, which is not the case for $\overline{C}(K)$, but the inverse limit $C'(K_\infty) = \varprojlim(C'(K_n) \otimes \mathbb{Z}_p)$ is of finite index in $\overline{C}(K_\infty)$ for any prime p including the case $p = 2$.

The purpose of this talk is to show that, for $p = 2$, there is an infinite family of real abelian fields K satisfying $C'(K_\infty) \neq \overline{C}(K_\infty)$.