GENERALIZED PUISEUX SERIES WITH NON-WELL-ORDERED SUPPORT AND THE PROBLEM OF LOCAL UNIFORMIZATION IN ARBITRARY CHARACTERISTIC.

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The main motivation for this work comes from the problem of resolution of singularities in algebraic geometry and its local variant, local uniformization. The problem of resolution of singularities asks whether, given an algebraic variety X over a field k, there exists a non-singular algebraic variety X' and a proper map $X' \to X$ which is one-to-one over the non-singular locus of X.

If we cover X' by affine charts, the problem becomes one of *parametrizing* pieces of X by small pieces of the Euclidean space k^n. Algebraically, this localized version of the problem can be interpreted as follows. Let R be k-algebra of finite type without zero divisors and let R_ν be a valuation ring containing R and having the same field of fractions as R. Find a *smooth* finite type k-algebra R' such that $R \subset R' \subset R_\nu$. The existence of such an R' is the **Local uniformization theorem**; it is known to hold when $char\ k = 0$ and is one of the central open problems in the field when $char\ k = p > 0$.

We will briefly recall the relevant notions from valuation theory and their connection to the problem of resolution of singularities and local uniformization.

Given a field L and a totally ordered abelian group Γ, the **field of generalized power series** $L\left(\left(t^\Gamma\right)\right)$ is, by definition, the ring of all the formal expressions $f = \sum\limits_{\gamma \in W(f)} a_\gamma t^\gamma$, where $a_\gamma \in L$ and $W(f)$ is some well ordered subset of Γ. The requirement that W be well ordered is needed in order to have a natural and well defined notion of multiplication in our field of generalized power series. We have the natural t-adic valuation defined on $L\left(\left(t^\Gamma\right)\right)$. A classical theorem of Irving Kaplansky from the nineteen fourties says that every valued field (K, ν) admits a (non-canoncical) injective homomorphism of valued fields to a suitable generalized power series field (this means, by definition, that the given valuation ν coincides with the restriction of the t-adic valuation to K). In this talk we will discuss a more general notion of **universal**

Puiseux series associated to ν (series which do not have well ordered support) as well as their possible applications to the prolem of Local uniformization in arbitrary characteristic.