

GENERIC OBJECTS

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An object from a given class can be called *generic* if it occurs typically in some building process involving smaller objects. The meaning of the word “typically” may be either probabilistic (occurring with probability one) or topological (occurring in a residual set). Yet another possibility for declaring an object to be generic is that it can be obtained through a winning strategy in an abstract Banach-Mazur game, where two players consecutively build a sequence of small structures leading to a structure which can be regarded as the limit of this sequence.

The theory of generic objects originates from model-theoretic setting explored by R. Fraïssé in 1954, motivated by Cantor’s theorem characterizing the set of rational numbers. Independently and much earlier, P. S. Urysohn constructed a universal ultrahomogeneous Polish space, which is now one of the classical examples of generic objects. It occurs with probability one as the completion of a natural random process building a sequence of finite metric spaces. Another example, soon after Fraïssé’s work, is the *random graph*, which is generic among all countable graphs and again occurs with probability one in a natural process of building a sequence of finite graphs.

Our aim is to describe a fairly general theory of generic objects in the framework of metric-enriched category theory. We will present several relevant examples, some of them coming from algebra, logic, topology, and functional analysis.