

# ON THE INTERSECTION GRAPH OF MODULES

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There are many studies of graphs associated to algebraic systems. One of the most important is the intersection graph of modules. For a given module  $M$  it is defined as the simple, undirected graph  $G(M)$  whose set of vertices consists of non-trivial submodules of  $M$  and two distinct submodules  $N_1, N_2$  are adjacent if and only if  $N_1 \cap N_2 \neq 0$ . For a given associative ring  $R$  the intersection graph of  $R$  is defined as  $G({}_R R)$ , where  ${}_R R$  denotes the left  $R$ -module  $R$ .

One of the first remarkable results in the area was obtained by D.Bertholf and G.Walls (*Graphs of finite abelian groups*. Czechoslovak Math.J.28(103), 1978, 365-368). In their paper an isomorphism problem on the intersection graph of finite abelian groups (regarded as modules over the ring of integers) was solved. From this result it in particular follows that if  $A, B$  are finite abelian  $p$ -groups for a prime  $p$ , then  $A \cong B$  if and only if  $G(A) \cong G(B)$ . This result does not extend to all modules or even all finite abelian groups but it shows that properties of the intersection graph of modules carry quite important information on modules and can be applied to study their structure.

The aim of the talk is to survey some results on the relationship between properties of a module  $M$  and the graph  $G(M)$ . They concern in particular the clique and chromatic numbers of  $G(M)$ . We will also characterize modules or rings with some given properties of their intersection graph and solve a problem raised by S.Akbari, H.A.Tavallaee and S.Khalashi Ghezalahmad (*Some results on the intersection graph of rings*. J.Algebra Appl.12, 2013, 1250200).

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