

ON THE CONGRUENCE

$$f(x) + g(y) + c = 0 \pmod{xy}$$

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L.J.Mordell (Acta Math.88, 1952, 77–83) stated the following theorem:

let $f(x) = a_0x^m + a_1x^{m-1} + \cdots + a_{m-1}x \in \mathbb{Z}[x]$, $g(y) = b_0y^n + b_1y^{n-1} + \cdots + b_{n-1}y \in \mathbb{Z}[y]$, $c \in \mathbb{Z}$. Then the congruence $f(x) + g(y) + c = 0 \pmod{xy}$ has infinitely many integer solutions x, y .

He proved it in the case $f(x) = ax^3$, $g(y) = by^3$, $|ab| > 1$, $c \neq 0$.

The lecture will be concerned with Mordell's assertion and a proof of the following theorem will be outlined.

If $m < 4$, $n = 1$ the congruence $f(x) + g(y) + c = 0 \pmod{xy}$ has infinitely many solutions, if and only if the equation $f(x) + g(y) + c = 0$ is soluble in integers.