

A GEOMETRIC MODEL OF AN ARBITRARY REAL CLOSED FIELD

Stanisław Spodzieja
University of Lodz (Łódź, Poland)

In the study of Hilbert's 17th problem, orderings of a real field k are of importance. By the Artin-Schreier theorem the study of such orderings amounts to considering real closures of k . The aim of this talk is to construct a universal model of an arbitrary real closed field. To this end, we construct, in terms of Nash functions, all real closures of the rational function fields $k = \mathbb{Q}(\Lambda_T)$, where $\Lambda_T = (\Lambda_t : t \in T)$ and $T \neq \emptyset$ is a system of any number of variables. This suffices to achieve our purpose, because any real closed field R is order-preserving isomorphic to a real closure of some field $\mathbb{Q}(\Lambda_T)$. If $T = \emptyset$, then $\mathbb{Q}(\Lambda_T) = \mathbb{Q}$, and the above is obvious.

We also give a characterization of any Archimedean field in terms of fields of Nash functions.