ON THE EXISTENCE OF $D(w)$-QUADRUPLES IN RINGS OF INTEGERS OF SOME NUMBERS FIELDS

Zrinka Franušić
University of Zagreb (Zagreb, Croatia)

Abstract

Let R be a commutative ring with unity 1 and $w \in R$. The set of nonzero and distinct elements $\{a_1, a_2, a_3, a_4\}$ in R such that $a_i a_j + w$ is a perfect square in R for $1 \leq i < j \leq 4$ is called a *Diophantine quadruple with the property* $D(w)$ in R or just a $D(w)$-*quadruple*. We consider the problem of existence of $D(w)$-quadruples in rings of integers of numbers fields. The conjecture says that there exists a $D(w)$-quadruple if and only if w can be represented as a difference of two squares, up to finitely many exceptions. The conjecture is shown to be true for the ring of integers $\mathbb{Z}$, the ring of Gaussian integers $\mathbb{Z}[i]$, rings of integers of some real quadratic fields $\mathbb{Q}(\sqrt{d})$ and the ring $\mathbb{Z}[\sqrt{-2}]$. The aim of this talk is to show that the conjecture is correct for the ring of integers of the pure cubic number field $\mathbb{Q}(\sqrt[3]{2})$ and the imaginary quadratic field $\mathbb{Q}(\sqrt{-3})$.