ON THE ARITHMETIC PROPERTIES OF STERN POLYNOMIALS

Maciej Gawron
Jagiellonian University (Kraków, Poland)

We investigate Stern polynomials defined by $B_0(t) = 0$, $B_1(t) = 1$, and for $n \geq 1$ by the recurrence relations $B_{2n}(t) = tB_n(t)$, $B_{2n+1}(t) = B_n(t) + B_{n+1}(t)$. We prove conjecture of Ulas that all possible rational roots of Stern polynomials are in the set $A = \{0, -1, -1/2, -1/3\}$. For $a \in A$ we investigate the set $R_a = \{n | B_n(a) = 0\}$. We prove that the set $R_a \cap (2\mathbb{N} + 1)$ is infinite, and R_a has lower density zero for $a = -1/2, -1/3$. The main tools in the proof are automatic sequences, and Frobenius-Perron Theorem. We give complete characterization of n such that $\deg(B_n) = \deg(B_{n+1})$ and $\deg(B_n) = \deg(B_{n+1}) = \deg(B_{n+2})$, resolving another conjecture of Ulas. Finally we study reciprocal Stern polynomials. We present two sequences $(u_n), (v_n)$ such that $B_n(t)$ is reciprocal for each n of the form $2^m - u_n, 2^m - v_n$. We give some remarks about the density of reciprocal Stern polynomials.