PYTHAGORAS NUMBERS OF FIELDS OF ALGEBRAIC FUNCTIONS AND LAURENT SERIES

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Wojciech Kucharz showed in the 90's that an algebraic function field of transcendence degree d over $\mathbb{R}$ (or any real closed base field) has pythagoras number at least $d+1$. We extend this lower bound to algebraic function fields over arbitrary formally real fields. In higher dimension, we obtain this by observing that the pythagoras number of any algebraic function field in $d \geq 2$ variables is lower bounded by that of a rational function field in $d-1$ variables over any finite extension of the base field that admits a place from the algebraic function field. The latter remains true (with the same proof) when replacing the words "function field in $d \geq 2$ variables" by the words "field of formal Laurent series in $d \geq 2$ variables". Time permitting, we will present a recent observation by Yong Hu that this lower bound together with a previous upper bound-result on the pythagoras of $k((t))(x)$ of ours (j.w. Becher and Van Geel) directly implies the equality $p(k((x,y))) = \sup\{\ell(x) \mid \ell/k \text{ finite field extension }\}$. In the special case $k = \mathbb{R}$ (or more generally when k hereditarily pythagorean), this was already shown by Choi, Dai, Lam and Reznick in their 1982 paper.