SUMS OF SQUARES IN NONREAL COMMUTATIVE RINGS

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Let R be a commutative ring with $1 \neq 0$. We call R nonreal if -1 is a sum of squares in R. We study invariants for nonreal rings pertaining to sums of squares. The following such invariants have been studied by various authors:

- the level $s(R)$, the least n such that -1 is a sum of n squares in R;
- the sublevel $\underline{s}(R)$, the least n such that 0 has a representation $0 = x_1^2 + \ldots + x_{n+1}^2$ such that the x_i generate R as an ideal;
- the Pythagoras number $p(R)$, the least n such that each sum of squares in R can be written as a sum of $\leq n$ squares.

We will introduce two new invariants, the metalevel s_m and the hyperlevel s_h. It turns out that $s_m = \underline{s}$, but our more "geometric" approach when defining s_m and s_h allows to get more precise relations between all these invariants. We will study certain generic rings that realize prescribed values for the various levels, and we will also give a survey on what is known about the possible 4-tuples that can be realized as (s_m, s_h, s, p) for nonreal rings.

This is joined work with David Leep (University of Kentucky)