DISCRETE UNIVERSALITY OF HURWITZ ZETA-FUNCTIONS

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It is well known that the Hurwitz zeta-function $\zeta(s,\alpha)$, $s=\sigma+it$, for certain values of the parameter α, $0<\alpha\leq 1$, is universal. This means that any analytic function can be approximated by shifts $\zeta(s+i\tau,\alpha)$, $\tau\in\mathbb{R}$ (continuous universality), or by shifts $\zeta(s+ikh,\alpha)$, $k\in\mathbb{N}_0$, $h>0$ is a fixed number (discrete universality). We consider the discrete universality for a new class of parameters α and h. Let $\mathcal{K}$ be the class of compact subsets of $D=\{s\in\mathbb{C}:\frac{1}{2}<\sigma<1\}$ with connected complements, and let $H(K)$, $K\in\mathcal{K}$, be the class of continuous functions on K which are analytic in the interior of K. Then we have the following statement.

Theorem. *S*uppose that the set $\left\{(\log(m+\alpha):m\in\mathbb{N}_0)\,,\frac{\pi}{h}\right\}$ is linearly independent over $\mathbb{Q}$. Let $K\in\mathcal{K}$ and $f(s)\in H(K)$. Then, for every ε,

$$\liminf_{N\to\infty}\frac{1}{N+1}\#\left\{0\leq k\leq N:\sup_{s\in K}|\zeta(s+ikh,\alpha)-f(s)|<\varepsilon\right\}>0.$$

For example, we can take $\alpha=\pi^{-1}$, $h=m^{-1}$, $m\in\mathbb{N}$.

Also, we will discuss the joint discrete universality for $\zeta(s)$ and $\zeta(s,\alpha)$ (discrete version of the Mishou theorem), where $\zeta(s)$ denotes the Riemann zeta-function, and for $\zeta(s,\alpha_1),,...,\zeta(s,\alpha_r)$.