

# MIXED JOINT UNIVERSALITY OF THE RIEMANN AND LERCH ZETA-FUNCTIONS

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Since a remarkable Voronin's work [1] on the universality of the Riemann zeta-function, it is known that the majority of other zeta and  $L$ -functions also are universal in the sense that their shifts approximate uniformly on compact subsets of certain regions a wide class of analytic functions. Also, a more complicated approximation property of zeta and  $L$ -functions – the joint universality – is known. In this case, we deal with a simultaneous approximation of a given system of analytic functions. The first result in this direction also is due to Voronin who obtained [2] the joint universality of Dirichlet  $L$ -functions. H. Mishou began to study the joint universality for zeta-functions having and having no Euler product over primes. He proved [3] a joint universality theorem for the Riemann zeta-function and Hurwitz zeta-function. The universality of such a type is called mixed joint universality and is investigated for various zeta functions, for example, in [4] this result has been generalized for the Riemann zeta-function and a collection of periodic Hurwitz zeta-functions, in [5] it has been extended for several periodic and periodic Hurwitz zeta-functions.

In the talk, we also will discuss a new result [6] that every system of analytic functions can be approximated simultaneously uniformly on compact subsets of some region by a collection consisting of shifts of the Riemann zeta-function and a collection of Lerch zeta functions (with parameters algebraically independent over the field of rational numbers), whose coefficients, in general, are not periodic.

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[2] S. M. Voronin, On the functional independence of Dirichlet  $L$ -functions, *Acta Arith.*, **27**, 493–503 (1975).

[3] H. Mishou, The joint value distribution of the Riemann zeta-function and Hurwitz zeta-functions, *Lith. Math. J.*, **47**(1), 32–47 (2007).

[4] J. Genys, R. Macaitienė, S. Račkauskienė, D. Šiaučiūnas, A

mixed joint universality theorem for zeta-functions, *Math. Modelling and Analysis*, **15**(4), 431–446 (2010).

[5] A. Laurinćikas, Joint universality of zeta-functions with periodic coefficients, *Izv. Ross. Akad. Nauk., Ser. Mat.*, **74**(3), 79–102 (2010).

[6] A. Laurinćikas, R. Macaitienė, Joint universality of the Riemann zeta-function and Lerch zeta-functions, *Nonlinear Analysis: Modelling and Control*, **18**(3), 314–326, 2013.