

PRODUCTS OF PFISTER FORMS

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Forms with diagonalization $\langle 1, a_1 \rangle \otimes \dots \otimes \langle 1, a_n \rangle$ for some scalars a_1, \dots, a_n , known as Pfister forms, play a central role in the theory of quadratic forms. The isotropy behaviour of their products with other forms has been the subject of long-standing interest. A classical result in this regard, established by Elman and Lam in the early seventies, states that, for q and π forms with π Pfister, the Witt index of $q \otimes \pi$ is a multiple of the dimension of π . As a consequence, if $q \otimes \pi$ is anisotropic over a field F , then its Witt index over the generic extension that makes it isotropic (its first Witt index) is at least the dimension of π .

We will establish a refinement of this result, determining that the first Witt index of $q \otimes \pi$ is bounded below by the product of the dimension of π and the first Witt index of q . Whereas this bound is not always attained, we will establish attainment for certain classes of forms. In particular, it readily follows from our bound that if q has the maximal splitting property, then $q \otimes \pi$ also has maximal splitting.

With respect to this and other phenomena that are preserved/accentuated by multiplication with a Pfister form, we will consider the possibility of “going-down”. We will outline some results in this direction in the case where the Pfister form is generated by variables, and will discuss some potential applications of these results to topics of interest within quadratic form theory.