

REAL SEMIGROUPS: SPECTRAL AND FANS

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Real semigroups (RS) were introduced in [DP], where it is proved that they constitute an “algebraic” functorial dual to the **abstract real spectra** introduced in Marshall [M].

After a brief review of the basics of RSs I’ll deal with two rich classes of them: **spectral real semigroups** (SRS) and **fans**.

SRSs come from the spectral topology of the character space of RSs. There is a functorial duality between SRSs and (arbitrary) spectral spaces whose specialization order is a root system. SRSs are also characterized as those RSs whose representation partial order is a distributive lattice. Every RSs has a “hull” which is a spectral RS. Examples: the RS’s associated to lattice-ordered rings are spectral.

I’ll also introduce a notion of fan for RSs, a natural generalization of the well-known notion of fan for reduced special groups and abstract spaces of orders. They turn out to be very natural objects possessing a very rich theory whose main features I will summarize. Examples: real valuation rings of fields verifying Bröcker’s trivialization theorem.

References

- [DP] M. Dickmann, A. Petrovich, *Real Semigroups and Abstract Real Spectra. I*, Contemporary Math. **344** (2004), 99-119.
[M] M. Marshall, **Spaces of Orderings and Abstract Real Spectra**, Lecture Notes Math. **1636**, Springer, 1996.