POLYNOMIAL CYCLES IN FIELDS OF UNIT RANK ONE.

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A finite tuple of distinct elements $\{x_0, x_1, ..., x_{n-1}\}$ of a ring R is called a polynomial cycle if there is a polynomial $f(X) \in R[X]$ such that $f(x_0) = x_1, f(x_1) = x_2, ..., f(x_{n-1}) = x_0$. The number n is called the length of that cycle, and $C(R)$ is the set of all possible lengths of polynomial cycles in R.

One can easily get $C(Z) = \{1, 2\}$.

In 1990-91 J.Boduch and G.Baron established $C(Z_K)$ for $K = Q(\sqrt{d})$-quadratic extensions of Q, with d squarefree. Namely, it equals :
$\{1, 2, 3, 6\}$ for $d = -3$;$\{1, 2, 3, 4\}$ for $d = 5$; $\{1, 2, 4\}$ for $d = -1, 2$; and $\{1, 2\}$ for other d.

In 2004 W.Narkiewicz found $C(Z_K)$ for all cubic fields K with negative discriminant D. Namely, it equals

$\{1, 2, 3, 4, 5\}$ for $D = -23$;$\{1, 2, 3, 4, 6\}$ for $D = -31$; $\{1, 2, 4\}$ for $D = -44, -59$; and $\{1, 2\}$ for all other D.

In the talk I want to establish $C(Z_K)$, for K of signature $(0, 2)$, hence (as in the preceding examples) the unit rank is ≤ 1, and other fields have unit rank at least 2. I will call such K trivial if $C(Z_K) = \{1, 2\}$, or $C(Z_K) = C(Z_{Q(\sqrt{2})})$ if $K \supset Q(\sqrt{2})$, or similar conditions with $i, \sqrt{-3}, \sqrt{5}$ instead of $\sqrt{2}$. The most interesting are non-trivial ones, and I will establish them (there are 17 of them, up to an isomorphism).

I also want to compare the problem to the several variables case, which (surprisingly) in many aspects turns out easier.