FINITELY CONTINUOUS DIFFERENTIALS ON GENERALIZED POWER SERIES

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A modern view on Jacobi's formula calls for notions of differentials and residues on a field with a valuation. We recall the formula: Let $t_{ij} \in \mathbb{Z}$ and $\Phi_1, \cdots, \Phi_n$ be Laurent series in variables $X_1, \cdots, X_n$ such that $\Phi_i X_1^{-t_{i1}} \cdots X_n^{-t_{in}}$ is a power series with nonzero constant term. For a Laurent series Ψ, Jacobi's formula asserts that the coefficients of $(X_1 \cdots X_n)^{-1}$ in $\Psi(\Phi_1, \cdots, \Phi_n) \det(\partial\Phi_i/\partial X_j)$ and in $\Psi \det(t_{ij})$ are the same. The appearance of the determinant $\det(t_{ij})$ in the formula is in fact a theme of logrithmic differentials, whose rigorous foundation is required for algebraic manipulations. Let κ be a field and $\mathcal{G}$ be a totally ordered Abelian group. We consider the field $\kappa[[e^{\mathcal{G}}]]$ (also denoted by $[[\kappa^{\mathcal{G},\geq}]]$ or $\kappa[[\mathcal{G}]]$ in the literature) of generalized power series with exponents in $\mathcal{G}$ and with coefficients in κ. Our notation $\kappa[[e^{\mathcal{G}}]]$ for the field of generalized power series emphasizes the interplay between logarithms and exponents while in accordance with the classical notation for formal power series. Given a subgroup $\mathcal{H}$ of $\mathcal{G}$ such that $\mathcal{G}/\mathcal{H}$ is finitely generated, we construct a vector space $\Omega_{\mathcal{G}/\mathcal{H}}$ of differentials as a universal object in certain category of $\kappa[[e^{\mathcal{H}}]]$-derivations on $\kappa[[e^{\mathcal{G}}]]$. The construction is based on M.-H. Mourgues' notion of finite convergence. We also define logarithmic residues, which together with $\Omega_{\mathcal{G}/\mathcal{H}}$, gives rise to a framework for Jacobi's formula and other combinatorial phenomena.