# ON THE WEIGHTED DENSITY OF IMAGE SETS OF POSITIVE INTEGERS UNDER A FUNCTION PRESERVING WEIGHTED DENSITY

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Denote by $\mathbb{N}$ the set of all positive integers, let $A \subset \mathbb{N}$ and let $f : \mathbb{N} \to (0, \infty)$ be a weight function. Let us denote

$$S_f(A, n) = \sum_{a \le n,\ a \in A} f(a), \qquad S_f(n) = \sum_{a \le n} f(a).$$

The weighted density of the set $A \subset \mathbb{N}$ with respect to the function $f$, called $f$-density, is defined as

$$d_f(A) = \lim_{n \to \infty} \frac{S_f(A, n)}{S_f(n)},$$

if this limits exists.

Nathanson and Parikh [*Density of sets of natural numbers and Lévy group*, J. Number Theory **1**24 (2007), 151–158] proved the following result (symbol $d(A)$ denotes the asymptotic density of the set $A$)

*Let $\pi : \mathbb{N} \to \mathbb{N}$ be an one-to-one function such that if the set $A$ of positive integers has asymptotic density, then the set $\pi(A)$ also has asymptotic density. Let $\lambda = d(\pi(\mathbb{N}))$. Then $d(\pi(A)) = \lambda d(A)$ for all $A$ having asymptotic density.*

The aim of this talk to generalize this result considering $f$-density instead of asymptotic density.