ON PROPERTIES OF SOME TYPE OF THE NUMBERS CONNECTED WITH GENERALIZED REPUNITS

Pavel Trojovský, Eva Hladíková
(University Hradec Kralove, Hradec Kralove)

A repunit R_n is any integer written in the form $R_n = (10^n - 1)/9$. Long interest was attended to finding a repunit primes. It's known that R_2, R_{19}, R_{23}, R_{317}, R_{1031} are primes and R_{49081}, R_{86453}, R_{109297}, R_{270343} are probable primes. Snyder extended the notation repunit to one in which for some integer $b \geq 2$ by this way $R_n(b) = (b^n - 1)/(b-1)$. They are called as *generalized repunits*. Some facts on the divisibility and primality of $R_n(b)$ was found and it was published many tables on their factorizations. For example for $b = 2$ these are the Mersenne numbers $M_n = 2^n - 1$. One of little known facts is the property, that $n \nmid M_n$ for any integer $n > 1$. In this paper we will study whether generalized repunits $R_n(k)$ have the similar property $n \nmid R_n(k)$. It is obvious that the numbers $R_n(k+1)$ are connected with the binomial theorem, thus $R_n(k+1) = \sum_{i=0}^{n-1} \binom{n}{i} k^{n-1-i}$. Similarily we can obtain numbers $J_n(k) = \sum_{i=0}^{n-2} \binom{n}{i} k^{n-2-i}$, where $k \geq 0$, $n \geq 2$ are any integers. In our talk some results about divisibility of $R_n(k)$ and $J_n(k)$ are stated. Further the convolution formulas for the numbers $R_n(k)$ and $J_n(k)$ are derived using their generating functions.