RAMANUJAN TYPE TRIGONOMETRIC FORMULAS FOR ARGUMENTS $\frac{2\pi}{7}$ AND $\frac{2\pi}{9}$

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The main aim of my work will be to present some new Ramanujan type trigonometric identities in the spirit of his original identities

$$\Big(\cos\frac{2\pi}{7}\Big)^{1/3}+\Big(\cos\frac{4\pi}{7}\Big)^{1/3}+\Big(\cos\frac{8\pi}{7}\Big)^{1/3}=\Big(\frac{5-3\sqrt[3]{7}}{2}\Big)^{1/3},$$
$$\Big(\cos\frac{2\pi}{9}\Big)^{1/3}+\Big(\cos\frac{4\pi}{9}\Big)^{1/3}+\Big(\cos\frac{8\pi}{9}\Big)^{1/3}=\Big(\frac{3\sqrt[3]{9}-6}{2}\Big)^{1/3}.$$

For example the following identity will be given

$$\sqrt[3]{\frac{\cos(\beta)}{\cos(2\beta)}}\big(2\cos(\beta)\big)^n+\sqrt[3]{\frac{\cos(2\beta)}{\cos(4\beta)}}\big(2\cos(2\beta)\big)^n+ \\ +\sqrt[3]{\frac{\cos(4\beta)}{\cos(\beta)}}\big(2\cos(4\beta)\big)^n= \\ =-\Bigg(\sqrt[3]{\frac{\cos(\beta)}{\cos(2\beta)}}\big(2\cos(2\beta)\big)^{n+1}+\sqrt[3]{\frac{\cos(2\beta)}{\cos(4\beta)}}\big(2\cos(4\beta)\big)^{n+1}+ \\ +\sqrt[3]{\frac{\cos(4\beta)}{\cos(\beta)}}\big(2\cos(\beta)\big)^{n+1}\Bigg)=\sqrt[3]{3}\,\Psi_n,$$

where $\beta=2\pi/9$, $\Psi_0=0$, $\Psi_1=3$, $\Psi_2=0$ and $\Psi_{n+3}-3\Psi_{n+1}+\Psi_n=0$, $n\in\mathbb{Z}$.