

# LEVELS AND SUBLEVELS OF RINGS

Detlev Hoffmann

(University of Nottingham, Nottingham)

The *level*  $s(R)$  of a ring  $R$  with  $1 \neq 0$  is defined to be  $\infty$  if  $-1$  is not a sum of squares in  $R$ . Otherwise,  $s(R)$  is the smallest  $n$  such that  $-1$  can be written as a sum of  $n$  squares in  $R$ . The *sublevel*  $\underline{s}(R)$  of  $R$  is defined to be the smallest  $n$  such that there exists a unimodular vector  $(x_1, \dots, x_{n+1}) \in R^{n+1}$  such that  $\sum_{i=1}^{n+1} x_i^2 = 0$ , provided such an  $n$  exists, otherwise one puts  $\underline{s}(R) = \infty$ . We give a survey of some known results about levels and sublevels of rings and also present some new results.