CURVES SINGULARITIES AND THE EGGERS TREES

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We consider a reduced series $f \in \mathbb{C}\{X,Y\}$ without constant term ($\mathbb{C}\{X,Y\}$ is the ring of convergent power series) and the germ $f = 0$ centered at $0 \in \mathbb{C}^2$. For $f, g \in \mathbb{C}\{X,Y\}$ we define the *intersection multiplicity* $(f,g)_0$ as the $\mathbb{C}$-codimension of the ideal generated by f and g in $\mathbb{C}\{X,Y\}$. For every irreducible f (a *branch*) we define the *semigroup* $\Gamma(f) = \{(f,g)_0 : g \in \mathbb{C}\{X,Y\}, g \notin (f)\}$. We call two germs $f = 0$ and $g = 0$ *equisingular* if there exist factorizations $f = f_1 \dots f_r$ and $g = g_1 \dots g_s$ into irreducible factors such that $r = s$, $\Gamma(f_i) = \Gamma(g_i)$ for $i = 1, \dots, r$ and $(f_i, f_j)_0 = (g_i, g_j)_0$ for $i, j = 1, \dots, r$. Equisingularity relation defines *equisingularity classes* in the space of germs. We represent every eqiusingularity class by the *Eggers tree* which is a graph with some decorations. In order to construct the tree we consider an *order of contact* of Płoski for irreducible f, g:

$$d(f,g) = \frac{(f,g)_0}{(\operatorname{ord} f)(\operatorname{ord} g)} .$$

This concact satisfy the *strong triangle property*

$$d(f,g) \geq \min\{d(f,h),\, d(g,h)\}.$$

Therefore the set $\mathcal{B}$ of all branches form an ultrametric space. As an application we describe the tree for nondegenerate germs (in the sence of Kouchnirenko). We develope a class of singularities with one chain in the tree. For this class we prove that the Jacobian-Newton polygon (introduced by Teissier) characterizes the equisingularity type assuming that the “colours” of edges of the Eggers tree are known.