DESINGULARIZATION OF QUASI-EXCELLENT SCHEMES OF CHARACTERISTIC ZERO

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Grothendieck proved in EGA IV that if any integral scheme of finite type over a locally noetherian scheme X admits a desingularization, then X is quasi-excellent (or qe), and conjectured that the converse is probably true. It was shown recently that this is indeed the case for qe schemes over $\mathbf{Q}$. Moreover, one can resolve any reduced qe scheme X over $\mathbf{Q}$ by a sequence $F(X)$ of blow ups with regular centers that sit over the singular locus of X, and one can construct F functorially in all regular morphisms $X' \to X$. The latter result is so strong that it formally (and easily) implies desingularization of many other objects, including analytic spaces (real, complex or p- adic) and formal qe schemes of characteristic zero. In particular, this gives a first proof that desingularization exists for formal schemes of finite type over $\mathbf{C}[[T]]$.

In this lecture I will explain the main desingularization results for qe schemes over $\mathbf{Q}$ and their corollaries. Then I will outline the main ideas of the proof, which runs by reducing to desingulairzation of varieties, which is now well understood due to works of Hironaka, Bierstone-Milman, Villamayor, Wlodarczyk, and others.